Problems Day 24, F 3/8/2024

Topic 11: Numerical methods for $y' = f(x, y)$, $y(x_0) = y_0$

Jeremy Orloff

Problem 1. Let $y' = x - y$, $y(1) = 3$.

(a) Use Euler's method to estimate $y(2)$ using 3 steps.

(b) Find the concavity of the integral curve at $x = 1$. Use this to predict if your estimate in Part (a) is too high or too low.

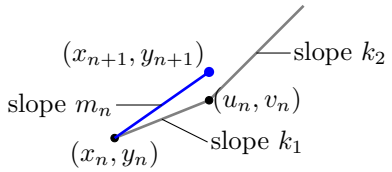
Problem 2. Challenge: Use Euler's method to estimate e.

–Pick your DE and IC

–Use various numbers of steps to estimate e.

Problem 3. Improved Euler is the following algorithm for choosing m_n.

- Fixed stepsize: Choose h at the start. It is the same for all steps.
- Choice of m_n: (see figure)



  $k_1 = f(x_n, y_n)$; (u_n, v_n) = regular Euler step: $u_n = x_n + h$, $v_n = y_n + k_1 h$.

  $k_3 = f(u_n, v_n)$

  $m_n = \dfrac{k_1 + k_2}{2}$

- Then (as always), $x_{n+1} = x_n + h$, $y_{n+1} = y_n + m_n h$.

Let $y' = y$, $y(0) = 1$. Estimate $y(1)$ using 3 steps and improved Euler.

ES.1803 Differential Equations

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