Problems Day 30, M 3/18/2024

Topic 14: Row reduction (day 1)

Jeremy Orloff

Problem 1. Let $A = \begin{bmatrix} 1 & 2 & 2 & 11 \\ 2 & 4 & 1 & 10 \\ 3 & 6 & 0 & 9 \end{bmatrix}$, $R = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(a) Use row reduction to show that $R = \text{RREF}(A)$.

(b) Identify the free and pivot variables of R.

(c) What is $\text{rank}(A)$?

(d) Give the relations between the free and pivot columns of R. (That is, write each free column as a linear combination of the pivot columns.)

(e) Verify that the columns of A have the same relations as in R.

(f) Use the relations between the columns of R to find a vector $\mathbf{v}$ such the $R\mathbf{v} = \mathbf{0}$.

(g) Verify that $A\mathbf{v} = \mathbf{0}$. ($\mathbf{v}$ your answer to the previous part)

(h) Find a solution to $A\mathbf{x} = \begin{bmatrix} 0 \\ 3 \\ 6 \end{bmatrix}$ by setting the free variabes to 0 and solving the resulting 3×2 system using row reduction on the augmented matrix.

ES.1803 Differential Equations

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