

## Problems Day 31, T 3/19/2024

Topic 14: Row reduction (day 2)

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**Problem 1.** Let  $A = \begin{bmatrix} 1 & 2 & 2 & 11 \\ 2 & 4 & 1 & 10 \\ 3 & 6 & 0 & 9 \end{bmatrix}$ .

We'll tell you that  $\text{RREF}(A) = R = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ .

- (a) Identify the free and pivot columns of  $R$  and  $A$ .
- (b) (i) Find a basis for  $\text{Null}(A)$ . Also, find  $\text{Null}(A)$ .  
(ii) How many elements are there in the basis? How many elements are there in  $\text{Null}(A)$ ?  
(iii) What is  $\dim(\text{Null}(A))$ ?  
(iv) Find  $\text{Null}(R)$ . (Hint: This requires no more work.)  
(v)  $\text{Null}(A)$  is a subspace of  $\mathbf{R}^n$ , what is  $n$ ?
- (c) (i) Find a basis of  $\text{Col}(A)$ . Also, find  $\text{Col}(A)$ .  
(ii) How many elements in the basis? In  $\text{Col}(A)$ ?  
(iii) What is  $\dim(\text{Col}(A))$ ? What is the rank of  $A$ ?  
(iv) Find  $\text{Col}(R)$ .  
(v) Does  $\text{Col}(A) = \text{Col}(R)$ ?

(d) Find a particular solution to  $A\mathbf{x} = \begin{bmatrix} 0 \\ 3 \\ 6 \end{bmatrix}$ . Do this by setting the free variables to 0 and solving the smaller system by row reduction.

(e) Give the general solution to  $A\mathbf{x} = \begin{bmatrix} 0 \\ 3 \\ 6 \end{bmatrix}$ .

**Problem 2.** Let  $R = \begin{bmatrix} 1 & 3 & 0 & 2 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ .

- (a) Give the relations between the free and pivot columns.
- (b) Find a matrix  $A = \begin{bmatrix} 2 & * & 6 & * \\ 5 & * & 1 & * \\ 7 & * & 2 & * \end{bmatrix}$  that has  $\text{RREF } R$ .

**Problem 3.** For the matrix  $A = \begin{bmatrix} 1 & 2 & 2 & 11 \\ 2 & 4 & 1 & 10 \\ 3 & 6 & 0 & 9 \end{bmatrix}$ , the pivot columns are Columns 1 and

3. Solve  $A\mathbf{x} = \begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix}$ .

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ES.1803 Differential Equations

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