Problems Day 35, M 4/1/2024

Topic 16: Eigenstuff (day 2)

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Problem 1. Suppose A is a 3×3 matrix with eigenvalue/vector pairs

$$\begin{array}{rcccc} \lambda & = & 2 & 3 & 5 \\ \mathbf{v} & = & \begin{bmatrix}1\\2\\0\end{bmatrix} & \begin{bmatrix}1\\1\\0\end{bmatrix} & \begin{bmatrix}0\\0\\1\end{bmatrix} \end{array}$$

(a) Give the general solution to $\mathbf{x}' = A\mathbf{x}$, $\left(\mathbf{x} = \begin{bmatrix}x\\y\\z\end{bmatrix}\right)$.

(b) Give the diagonalized form of A

(c) Give A^5, A^{-1}, $\det(A)$.

(d) Give a change of variable that decouples the system in Part (a). Write the decoupled system in matrix form and solve it.

Problem 2. Repeat problem (1) for the matrix $\begin{bmatrix}5 & 3\\1 & 3\end{bmatrix}$.

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