

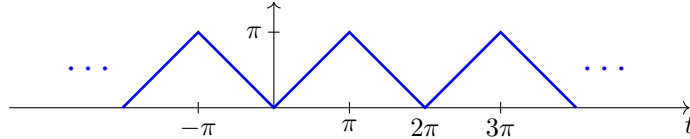
Problems Day 44, F 4/12/2024

Topic 21: Fourier series (day 2)

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Note: There is a useful integral table on the last page.

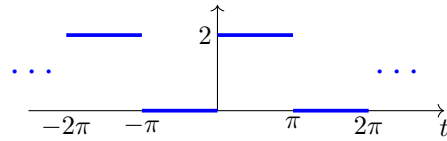
Problem 1. $\text{tri}(t) = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n \text{ odd}} \frac{\cos(nt)}{n^2}.$



Graph of $\text{tri}(t)$

- (a) Convince yourself that $\text{tri}'(t) = \text{sq}(t)$.
- (b) Check that the derivative of the Fourier series for $\text{tri}(t)$ equals the Fourier series of $\text{sq}(t)$.

Problem 2. Let $f(t) = 1 + \text{sq}(t)$. Find the Fourier series for $f(t)$.



Graph of $f(t)$

Problem 3. Let $g(t)$ have period 2 and $g(t) = \begin{cases} -1 & \text{for } -1 < t < 0 \\ 1 & \text{for } 0 < t < 1. \end{cases}$

Find the Fourier series for $g(t)$.

Integrals (for n a positive integer)

$$1. \int t \sin(\omega t) dt = \frac{-t \cos(\omega t)}{\omega} + \frac{\sin(\omega t)}{\omega^2}.$$

$$2. \int t \cos(\omega t) dt = \frac{t \sin(\omega t)}{\omega} + \frac{\cos(\omega t)}{\omega^2}.$$

$$3. \int t^2 \sin(\omega t) dt = \frac{-t^2 \cos(\omega t)}{\omega} + \frac{2t \sin(\omega t)}{\omega^2} + \frac{2 \cos(\omega t)}{\omega^3}.$$

$$4. \int t^2 \cos(\omega t) dt = \frac{t^2 \sin(\omega t)}{\omega} + \frac{2t \cos(\omega t)}{\omega^2} - \frac{2 \sin(\omega t)}{\omega^3}.$$

$$1'. \int_0^\pi t \sin(nt) dt = \frac{\pi(-1)^{n+1}}{n}.$$

$$2'. \int_0^\pi t \cos(nt) dt = \begin{cases} \frac{-2}{n^2} & \text{for } n \text{ odd} \\ 0 & \text{for } n \neq 0 \text{ even} \end{cases}$$

$$3'. \int_0^\pi t^2 \sin(nt) dt = \begin{cases} \frac{\pi^2}{n} - \frac{4}{n^3} & \text{for } n \text{ odd} \\ \frac{-\pi^2}{n} & \text{for } n \neq 0 \text{ even} \end{cases}$$

$$4'. \int_0^\pi t^2 \cos(nt) dt = \frac{2\pi(-1)^n}{n^2}$$

If $a \neq b$

$$5. \int \cos(at) \cos(bt) dt = \frac{1}{2} \left[\frac{\sin((a+b)t)}{a+b} + \frac{\sin((a-b)t)}{a-b} \right]$$

$$6. \int \sin(at) \sin(bt) dt = \frac{1}{2} \left[-\frac{\sin((a+b)t)}{a+b} + \frac{\sin((a-b)t)}{a-b} \right]$$

$$7. \int \cos(at) \sin(bt) dt = \frac{1}{2} \left[-\frac{\cos((a+b)t)}{a+b} + \frac{\cos((a-b)t)}{a-b} \right]$$

$$8. \int \cos(at) \cos(at) dt = \frac{1}{2} \left[\frac{\sin(2at)}{2a} + t \right]$$

$$9. \int \sin(at) \sin(at) dt = \frac{1}{2} \left[-\frac{\sin(2at)}{2a} + t \right]$$

$$10. \int \sin(at) \cos(at) dt = -\frac{\cos(2at)}{4a}$$

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ES.1803 Differential Equations

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