Problems Day 60, T 5/7/2024

Topic 30: Population models

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Problem 1. Fancier predator-prey (we've looked at this system once before):

$$\begin{aligned} x' &= 3x - x^2 - xy \\ y' &= y - y^2 + xy \end{aligned}$$

(a) Which is predator? prey?

(b) If there is no y ($y = 0$), what does the model say about x?

What is the model called in this case?

(c) Here is a table of critical points

Critical points	(x_0, y_0) :	$(0,0)$	$(0,1)$	$(3,0)$	$(1,2)$
Jacobian	$J(x_0,y_0)$:	$\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 2 & 0 \\ 1 & -1 \end{bmatrix}$	$\begin{bmatrix} -3 & -3 \\ 0 & 4 \end{bmatrix}$	$\begin{bmatrix} -1 & -1 \\ 2 & -2 \end{bmatrix}$
Eigenvalues	λ :	$3, 1$	$2, -1$	$-3, 4$	$\dfrac{-3 \pm \sqrt{7}\, i}{2}$
Eigenvectors (if needed) :			$\begin{bmatrix} 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ -7 \end{bmatrix}$	

Draw a phase portrait.

(d) Tell a story.

Problem 2. (Armand and Babette go nonlinear (Pset 9))
We won't copy down their story. The system is

$$\begin{aligned} x' &= x - 2y + \frac{1}{4}x^2 \\ y' &= 5x - y - y^2. \end{aligned}$$

(a) Find the critical points. (Hint: you'll end up with a quartic polynomial. One root is 0, another is a positive integer ≤ 5.)

(b) Linearize at each critical point and sketch the phase portrait of the nonlinear system.

(c) Interpret the results in terms of their relationship.