

Problems Day 7, F 3/1/2024

Topic 4: Complex numbers (day 2 of 2)

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Continue from Topic 4, Day 6.

Problem 1. Let $z_1 = 2 + 5i$, $z_2 = 1 + 3i$

(a) Compute $z_1 + z_2$, $z_1 \cdot z_2$, $z_1 \cdot \bar{z}_1$, $|z_1|$, $\text{Arg}(z_1)$.

(b) Find $\text{Re}(z_1)$, $\text{Im}(z_1)$.

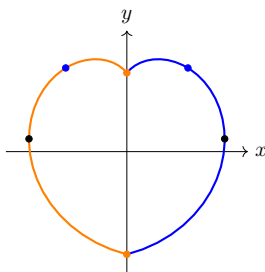
(c) Let $z = x + iy$. Compute $z \cdot \bar{z}$.

Problem 2. Find r and $\theta = \text{Arg}(z)$ for $z = i$, $z = 1 - i$, $z = 5(1 - i)$.

Problem 3.

(a) Write $\frac{i}{2}$ in polar form.

(b) Consider the diagram



Multiply the diagram by $i/2$, i.e., sketch the resulting image.

Problem 4. Show $\overline{e^{i\theta}} = e^{-i\theta}$.

Problem 5. Compute $(1 + \sqrt{3}i)^{10}$. (Use polar form.)

Problem 6. Compute $I = \int e^x \cos(5x) dx$.

Problem 7.

(a) Find the fifth roots of 1. Draw a picture.

(b) Find the fifth roots of $1 + i$. Draw a picture.

Problem 8.

(a) Draw the trajectory of $z = e^{it}$

(b) Draw the trajectory of $z = te^{it}$

(c) Plot the points $e^{ij\pi/4}$, for $j = 0, 1, 2, 3 \dots$

Problem 9.

(a) Write $\sin t$ and $\cos t$ in terms of e^{it} and e^{-it} .

(b) Find all real-valued functions of the form $f(t) = c_1 e^{it} + c_2 e^{-it}$, where c_1, c_2 are complex constants.

Problem 10. Find all the roots of $x^4 + x^2 = 0$.

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