ES.1803 Practice Questions – Quiz 3, Spring 2024
Covers Topics 9

This will probably take longer than 1 hour; the actual quiz will be considerably shorter. Not every possible question is covered. For more questions, you can also look at the in-class problems, problem section worksheets and psets.

Be wise in carrying out computations. If you get a problem to a point where you are sure –*emphasize sure*– you can carry out the computation, then stop and go on to the next problem. At the end you can check the solutions to see that you had correctly set up the computation.

Problem 1.
Consider the lightly damped second-order system $y'' + 2y' + 10y = \sin(\omega t)$.

For this problem, consider $\sin(\omega t)$ to be the input to the system. **(a)** What is the natural frequency of this system?

(b) If the damping is removed, what *integer* frequency ω_{int} will make the amplitude of $y_p(t)$ relatively large?

(c) Find any practical resonant frequencies for the damped system.

Problem 2.
Consider the DE $x'' + bx' + 5x = \cos(\omega t)$. Assume, for this system, that $\cos(\omega t)$ is the input.

(a) For what b is it possible for this system to undergo pure resonance?

(b) For all values of b in your answer to Part (a), give the corresponding resonant frequency.

(c) For each of the b values in the answer to Part (a), solve the DE with ω equal to the resonant frequency.

(d) Graph the amplitude response of each of the systems that can undergo resonance.

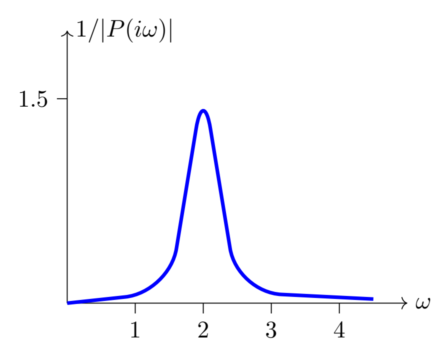
(e) Graph one nicely chosen solution from Part (c).

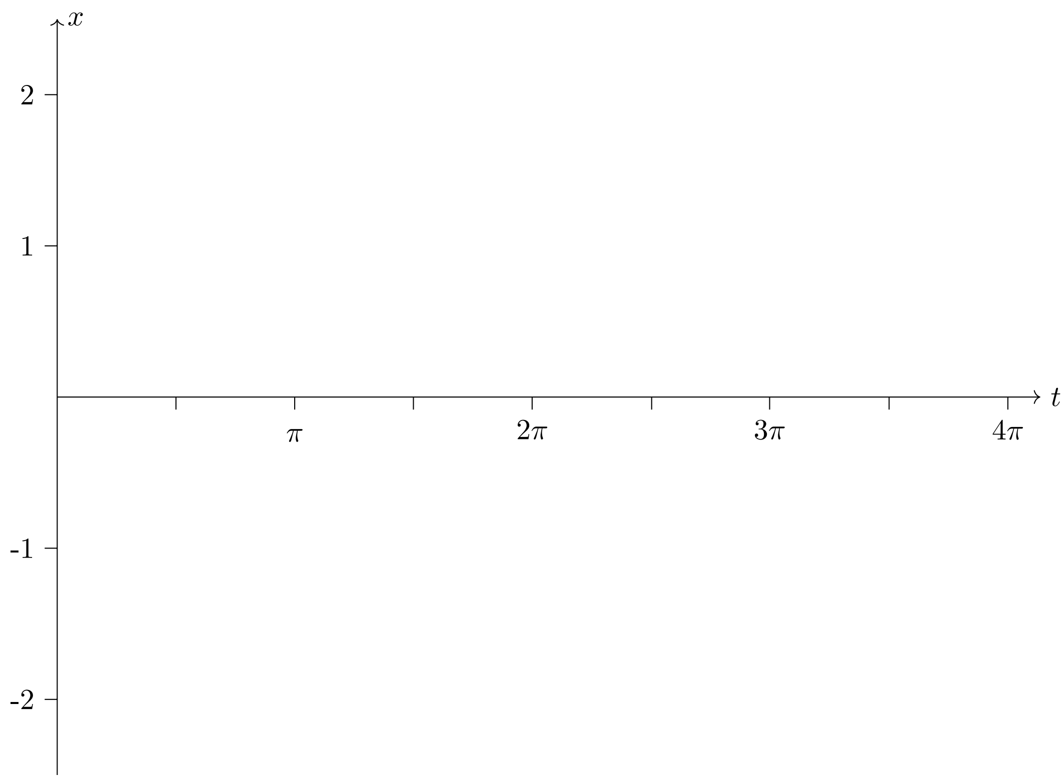
(f) (Unrelated to the DE above.) Assume we have a (possibly frictionless) physical system modeled by a second-order constant coefficient linear DE with positive coefficients. What criteria must the roots satisfy if this system can undergo pure resonance?

Problem 3.
Suppose that $P(r)$ is a polynomial and consider the DE

$$P(D)x = \cos t + \cos 2t + \cos 3t.$$

The graph of $\dfrac{1}{|P(i\omega)|}$ is shown. On the axes provided give a rough sketch of the periodic solution $x_p(t)$ to the DE. Give a brief explanation of your reasoning.





Problem 4.
The differential operator for this problem is $P(D) = D^2 + 4D + 13I$.

(a) (12) Find the general *real-valued* solution to $P(D)x = 2\cos(\omega t)$.

(b) (5) Assume that $\cos(\omega t)$ is the input to the system in Part (a). Give the amplitude response (as a function of ω).

We'll help you by saying that there is exactly one (practical) resonant frequency, which is at $\omega = \sqrt{5}$. Use this to help sketch a graph of the amplitude response. Be sure to label your axes.

Problem 5. Consider the forced undamped system: $x'' + 8x = \cos(\omega t)$.

(a) Why is this called a forced undamped system?

(b) Use the exponential response formula to find a solution.

(c) Consider the right-hand side of the DE to be the input and graph the amplitude response function.

(d) What is the resonant frequency of the system?

(e) Why is this called the natural frequency?

Problem 6.
Consider the forced damped second-order system: $x'' + 2x' + 9x = \cos(\omega t)$.

(a) What is the natural frequency of the system?

(b) Find the response of the system in amplitude-phase form.

(c) Consider the right-hand side of the DE to be the input. What is the amplitude response of the system?

(d) What is the practical resonant frequency?

(e) When $\omega = \sqrt{7}$, by how many radians does the output peak lag behind the input peak?

(f) For the forced undamped system $x'' + 9x = \cos(\omega t)$, give a detailed description of the phase lag for different input frequencies?

Problem 7. Consider the driven first-order system: $x' + kx = kF_0 \cos(\omega t)$.

Consider the input to be $F_0 \cos(\omega t)$. Solve the DE and find the amplitude response. Show there is never practical resonance.

End of practice quiz