Mechanics of Simulation

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Organization

- Presentation is
- Based on text
- Plus some Sensitivity Analysis

What is Simulation?

- Replicates outcomes of uncertain process (often called "Monte Carlo" simulation)
- It provides a way to describe what may occur over range of possible scenarios
- Can use variety of distributions:
  - regular or
  - irregular
  - continuous or not

Use of Simulation: not widespread New in General Practice

- Modern software makes simulation practical
  - 1000's of repetitions in seconds
- Model often simple, e.g., spreadsheet of profits
  - Simple example: Excel Add-in
  - Market versions: Crystal Ball®; @Risk® …
  - See Antamina case (other slide deck)

Requirements for Simulation

- Distributions for Key parameters
  - May be observed, estimated, assumed, or guessed
- Examples:
  - Observed: Rainfall, river flows over years
  - Estimated: Technical Cost Models
  - Assumed: Market data (historical prices)
  - Guessed: Judgment (ore quantity, quality)

Recommended Process

Steps:

1. Produce a standard valuation model
2. Do Sensitivity Analysis
   - One variable at a time
   - Probabilistically
3. Model System Performance
   - No Flexibility
   - With alternative flexibilities

Step 1: Produce Valuation Model

Parking Garage Case

- **Garage in area where population expands**
- **Inspired by Bluewater Mall in SE England**

- **Actual demand is necessarily uncertain**

- **Question (for this step): How do we value?**

Parking Garage Case details

- **Demand**
  - **At start is for 750 spaces**
  - **Over next 10 years is expected to rise exponentially by another 750 spaces**
  - **After year 10 may be 250 more spaces**
  - **could be +/- 50% off the projections**
  - **Annual volatility for growth is 10%**
- **Average annual revenue/space = $10,000**
- **The discount rate is taken to be 12%**

Parking Garage details (Cont)

- **Costs**
  - **annual operating costs (staff, cleaning, etc.) = $2,000 /year/space available**
  - **spaces used often < spaces available**
  - **Annual lease of the land = $3.6 Million**
  - **construction cost = $16,000/space + 10% for each level above the ground level**

- **Site can accommodate 200 cars per level**

Valuation Model

SYSTEM PARAMETERS			
Capacity per level	200	cars	
Number of levels	6	levels	[DESIGN PARAMETERS]

PERFORMANCE CALCULATION					
Year	**0**	**1**	**2**	**…**	**15**
Demand		750	893	…	1,634
Capacity		1,200	1,200	…	1,200
Revenue		7.5	8.9	…	12.0
Operating costs	0.0	3.6	3.6	…	3.6
Land leasing and fixed costs	3.3	3.3	3.3	…	3.3
Cashflow	-3.3	0.6	2.0	…	5.1
Discounted cashflow	-3.3	0.5	1.7	…	1.2
Present value of cashflow	26.7				
Capacity cost for up to two levels	6.8				
Capacity costs for levels above 2	17.4				
Net present value	2.5				

Figure D.1 Flexibility in Engineering Design de Neufville and Scholtes MIT Press 2011

Step 2: Sensitivity Analysis

- **One dimensional**

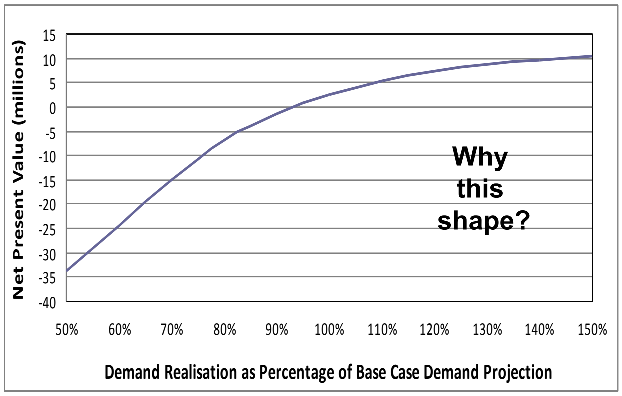


Figure D.2 Flexibility in Engineering Design de Neufville and Scholtes MIT Press 2011

Tornado Diagram

INPUT VARIABLE	Base case	%-deviation in Tornado diagram	
Demand deviation from projection	100%	50	%
Average annual revenue	$10,000	20	%
Average operating costs	$3,000	15	%
Land lease and other fixed costs	$3,330,000	5	%
Capacity cost	$13,600	10	%
Growth of capacity cost above level 2	10%	20	%
Average capacity utilization	80%	10	%

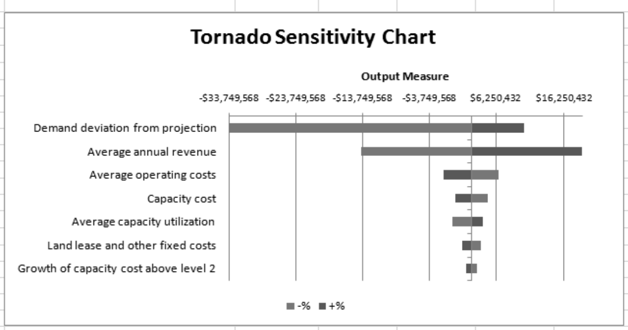


Figure D.3 Flexibility in Engineering Design de Neufville and Scholtes MIT Press 2011

Interpreting Sensitivity Analysis

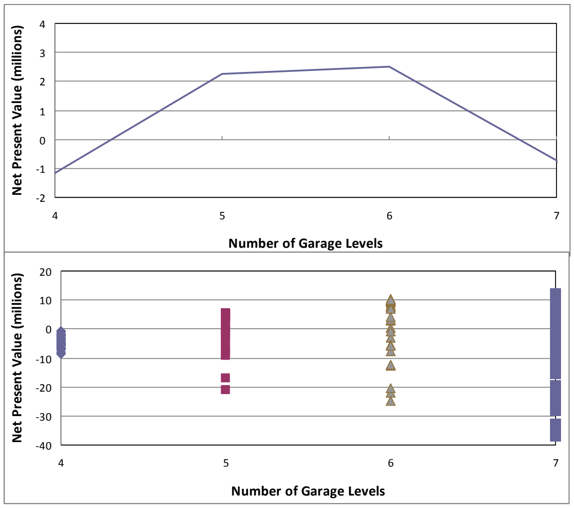
- Everything is uncertain – what should we pay attention to?
- Sensitivity analysis helps to answer this question
- In this case, the level of demand seems to affect performance (here, the value of investment) most.
- Such observations drive rest of analysis

Probabilistic Sensitivity Analysis

Trial	Demand deviation	Average annual revenue	Average operating costs	Land lease and other fixed costs	Capacity cost	Growth of cap cost above 2 levels	Average capacity utilization	NPV
1	134%	$11,016	$3,140	$3,495,435	$12,399	9.88%	79.69%	$17,625,663
2	141%	$11,852	$3,260	$3,304,375	$13,795	9.84%	79.06%	$23,221,598
3	86%	$10,193	$3,312	$3,430,938	$13,068	10.43%	79.50%	-$4,718,694
4	83%	$9,055	$3,283	$3,354,325	$14,036	9.90%	76.09%	-$15,456,972
5	83%	$10,114	$3,047	$3,316,798	$12,685	11.52%	87.40%	-$3,934,426
6	62%	$8,111	$3,283	$3,206,261	$13,100	11.02%	84.27%	-$36,456,858
7	76%	$10,515	$2,647	$3,386,862	$14,260	11.81%	83.77%	-$6,799,900
8	68%	$8,343	$3,440	$3,269,874	$14,830	9.71%	78.82%	-$32,839,236
9	114%	$10,272	$3,291	$3,410,315	$14,852	10.11%	85.44%	$5,120,553
10	68%	$10,151	$3,413	$3,437,877	$14,110	10.78%	85.28%	-$24,320,813
11	68%	$9,664	$3,153	$3,163,520	$13,841	11.51%	82.62%	-$21,314,749
12	129%	$11,189	$2,636	$3,430,040	$14,713	10.59%	75.51%	$16,118,317
13	132%	$11,832	$2,695	$3,179,077	$14,469	11.09%	84.10%	$30,373,102
14	114%	$10,250	$2,586	$3,189,684	$14,311	9.34%	75.61%	$9,916,978
15	105%	$9,754	$2,641	$3,218,399	$14,147	9.14%	72.32%	$2,261,573
16	63%	$9,595	$3,089	$3,240,407	$13,222	9.92%	77.66%	-$22,983,479
17	147%	$10,881	$2,702	$3,374,710	$13,140	10.25%	79.15%	$20,546,268
18	91%	$8,783	$3,275	$3,222,284	$12,731	9.97%	86.09%	-$10,350,864
19	67%	$9,320	$3,182	$3,477,948	$14,446	9.57%	81.29%	-$26,886,841
20	67%	$8,961	$3,302	$3,206,613	$14,770	9.87%	75.37%	-$26,433,140

Figure D.4 – variables moving together, not one at a time

Choice of best design in Context of Uncertainty



Figures D.8 and D.9

Flexibililty in Engineering Design

Top is Expected NPV, Bottom Graph shows range of outcomes.

Notice the difference in scale between the two graphs!

In this case, the uncertainties may dominate the choice.

Step 3: Introduce dynamic scenarios

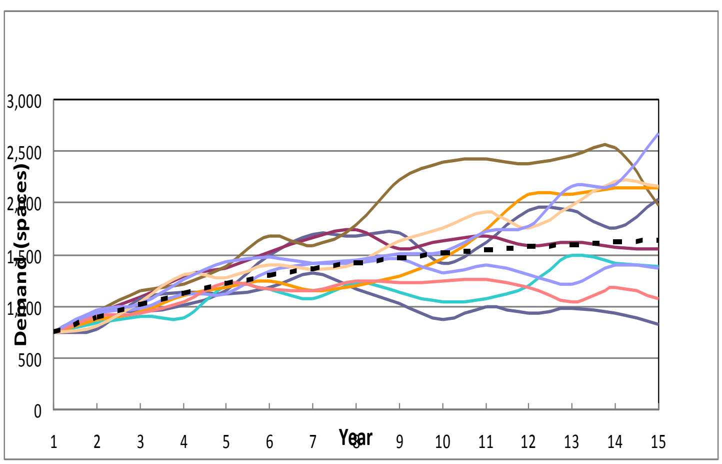


Figure D.6 Flexibility in Engineering Design de Neufville and Scholtes MIT Press 2011

Simulation: no flexibility

- Generate a dynamic scenario and apply it to model, period by period
- Calculate the performance in each period and over life of project (such as NPV)
- Report for many scenarios (1000 or more) representing distribution of uncertainty
- Calculate mean (such as ENPV) of results, also distribution

Shape of output distributions

- Traditional (NPV) analysis is:
  "numbers in, numbers out"
- Uncertainty analysis is:
  "shapes in, shapes out"
- "Target curve" is useful representation of "shape out." It is its cumulative distribution.

Target Curve

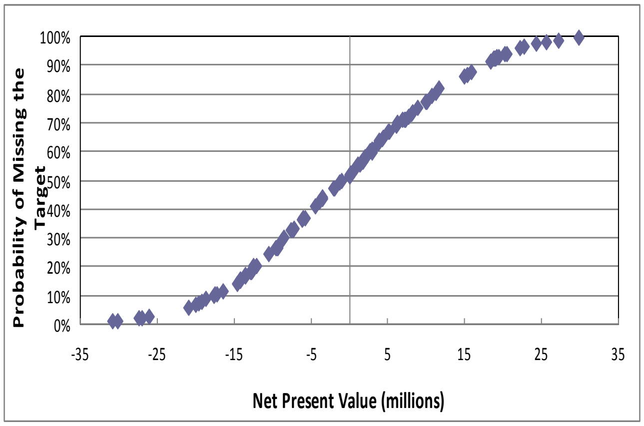


Figure D.14 Flexibility in Engineering Design de Neufville and Scholtes MIT Press 2011

Creation of Target Curve

- At end of Simulation, we have many trials
- What is probability of each?
- They are equal !!
- How do we get pdf?
- "Binning" the outcomes by ranges (= bins)
  - Percent samples in bin = P(outcome in bin)
- Note: We look at all, 1000s, of simulations!
  - But binning process easily automated

Simulation: with flexibility

- First question is: what kind of flexibility should we consider?
- Answer flows from the uncertainties that impact performance most
- In garage case, demand for spaces is most significant, so what kind of flexibility?
- For this case, we want flexibility as to size, or capacity of project – the number of levels to garage

Rules for exercising flexibility (such as option to close mine – Chile)

- When should we exercise flexibility?
- In simulation, this time cannot be calculated
- Why?
- Because number of possible future paths, states are too large to be searched
- Procedure: set up a priori conditions for when to exercise flexibility. These are "decision rules" for exercising flexibility

Example of "Decision Rules"

Consider a possible rule for Parking Garage

- "Expand if, over 2 years, observed demand is greater than capacity "
- Why would this make sense?
  => Because, want some assurance that growth is 'permanent'
- How could this be improved?
  => Change rule toward end of life? No addition in last 5 years?

Simulation WITH flexibility

- As for inflexible case, BUT
- At each period, check if decision rule for that period tells you to exercise flexibility – if yes, do so and take advantage of result
- Note Carefully: CAN EXERCISE MANY FLEXIBILITIES in same analysis! This is not possible in conventional financial analyses of options

Value of Flexibility by Simulation

1. Get distribution of consequences for plan or design without flexibility
   => NPV pdf, EV(NPV); Target Curve
2. Repeat above, considering availability of flexibility, and exercise at desired times
   => EV(NPV); Target Curve
3. Flexibility Value = difference of EV(NPV)s
   Compare Target Curves to see source of value