In this segment, we consider the sum of independent Poisson random variables, and we establish a remarkable fact, namely that the sum is also Poisson.

This is a fact that we can establish by using the convolution formula.

The PMF of the sum of independent random variables is the convolution of their PMFs.

So we can take two Poisson PMFs, convolve them, carry out the algebra, and find out that in the end, you obtain again a Poisson PMF.

However, such a derivation is completely unintuitive, and does not give you any insight.

Instead, we will derive this fact by using our intuition about what Poisson random variables really represent.

We will work with a Poisson process of rate lambda equal to 1.

But let us remind ourselves of the general Poisson PMF if we have a more general rate lambda.

This is the PMF of the number of arrivals in a Poisson process with rate lambda during a time interval of length tau.

And this Poisson PMF has a mean equal to lambda tau.

And you can think of lambda tau as being the parameter of this Poisson PMF.

So we say that this is a Poisson PMF with parameter equal to lambda times tau.

Now, let us consider two consecutive time intervals in this processes that have length mu and nu.

And let us consider the numbers of arrivals during each one of these intervals.

So we have M arrivals here and N arrivals there.

Of course, M and N are random variables.

What kind of random variables are they?

Well, the number of arrivals in the Poisson process of rate 1, over a period of duration mu is going to have a Poisson PMF in which lambda is one, tau, the time interval is equal to mu, so it's going to be a Poisson random variable with parameter, or mean, equal to mu.

Similarly for N, it's going to be a Poisson random variable with parameter equal to nu.

Are these two random variables independent?

Of course they are.

In a Poisson process, the numbers of arrivals in disjoint time intervals are independent random variables.

What kind of random variable is their sum?

Their sum is the total number of arrivals during an interval of length mu plus nu, and therefore this is a Poisson random variable with mean equal to mu plus nu.

So, what do we have here?

We have the sum of two independent Poisson random variables, and that sum turns out also to be a Poisson random variable.

More generally, if somebody gives you two independent Poisson random variables, you can always think of them as representing numbers of arrivals in disjoint time intervals, and therefore by following this argument, their sum is going to be a Poisson random variable.

And this is the conclusion that we wanted to establish.

It's a remarkable fact.

It's similar to the fact that we had established for normal random variables.

The sum of independent normal random variables is also normal, so Poisson and normal distributions are special in this respect.

This is a property that most other distributions do not have, with very few exceptions.