

Let us now look at some examples of sample spaces.

Sample spaces are sets.

And a set can be discrete, finite, infinite, continuous, and so on.

Let us start with a simpler case in which we have a sample space that is discrete and finite.

The particular experiment we will be looking at is the following.

We take a very special die, a tetrahedral die.

So it's a die that has four faces numbered from 1 up 4.

We roll it once.

And then we roll it twice [again].

We're not dealing here with two probabilistic experiments.

We're dealing with a single probabilistic experiment that involves two rolls of the die within that experiment.

What is the sample space of that experiment?

Well, one possible representation is the following.

We take note of the result of the first roll.

And then we take note of the result of the second roll.

And this gives us a pair of numbers.

Each one of the possible pairs of numbers corresponds to one of the little squares in this diagram.

For example, if the first roll is 1 and the second is also 1, then this particular outcome has occurred.

If the first roll is 2 and the second is a 3, then this particular outcome occurs.

If the first roll is a 3 and then the next one is a 2, then this particular outcome occurs.

Notice that these two outcomes are pretty closely related.

In both cases, we observe a 2 and we observe a 3.

But we distinguish those two outcomes because in those two outcomes, the 2 and the 3 happen in different order.

And the order in which they appear may be a detail which is of interest to us.

And so we make this distinction in the sample space.

So we keep the (3, 2) and the (2, 3) as separate outcomes.

Now this is a case of a model in which the probabilistic experiment can be described in phases or stages.

We could think about rolling the die once and then going ahead with the second roll.

So we have two stages.

A very useful way of describing the sample space of experiments-- whenever we have an experiment with several stages, either real stages or imagined stages.

So a very useful way of describing it is by providing a sequential description in terms of a tree.

So a diagram of this kind, we call it a tree.

You can think of this as the root of the tree from which you start.

And the endpoints of the tree, we usually call them the leaves.

So the experiment starts.

We carry out the first phase, which in this case is the first roll.

And we see what happens.

So maybe we get a 2 in the first roll.

And then we take note of what happened in the second roll.

And maybe the result was a 3.

So we follow this branch here.

And we end up at this particular leaf, which is the leaf associated with the outcome 2, 3.

Notice that in this tree we once more have a distinction.

The outcome 2 followed by a 3 is different from the outcome 3 followed by a 2, which would correspond to this particular place in the diagram.

In both cases, we have 16 possible outcomes.

4 times 4 makes 16.

And similarly, if you count here, the number of leaves is equal to 16.

The previous example involves a sample space that was discrete and finite.

There were only 16 possible outcomes.

But sample spaces can also be infinite.

And they could also be continuous sets.

Here's an example of an experiment that involves a continuous sample space.

So we have a rectangular target which is the unit square.

And you throw a dart on that target.

And suppose that you are so skilled that no matter what, when you throw the dart, it always falls inside the target.

Once the dart hits the target, you record the coordinates  $x$  and  $y$  of the particular point that resulted from your dart throw.

And we record  $x$  and  $y$  with infinite precision.

So  $x$  and  $y$  are real numbers.

So in this experiment, the sample space is just the set of  $x, y$  pairs that lie between 0 and 1.