All right.

So let us revisit the example from last lecture.

So we have a Markov chain with two states, one and two, and this Markov chain has a single recurrent class.

All right.

And then also it's not periodic right, because we have self transition of this type.

So as a result, this is well defined and these steady state probabilities from 1 to m, in that case for us, m = 2, right?

So let us write the system and solve the system of linear equation for this example here.

So what we have is pi 1 equals pi 1 times 0.5 plus pi 2 times 0.2.

So that's the first equation that corresponds to j equals 1.

Now, for j equals 2, pi 2 equals pi 1 times 0.5 plus pi 2 times 0.8.

So we have a system of two equations with two unknowns, pi 1 and pi 2.

Let us rewrite them, I pass this one on this side and this one on this side.

So we get pi 1 times 1 minus 0.5 minus 0.5 equals pi 2 times 0.2.

And this one pi 2 times 1 minus 0.8 is 0.2 equals pi 1 times 0.5.

We realize that these two happen to be the same, so they are not enough to define a unique solution, so we have to add another equation, and we know that these are probabilities.

So pi 1 plus pi 2 has to be one, and so now we're going to keep one of these two, let's say this one, I'm going to write it here.

And we can rewrite it by saying that pi 1 times 1/2 equals pi 2 times 1/5.

So now, we're going to take that, replace pi 1 equals 2/5 of pi 2 is the result of that.

And we're going to use that pi 1 and replace it here.

So we end it by 2 times 2/5 plus 1 equals 1, which means that from here, we get that pi 2 equals 5 plus 2/7 so 5/7, and then we use that and place it here and we end up having pi 1 equals 2/5 times 5/7 equals 2/7, and we check 5 plus 2 equals 7, so these are real probabilities.

So the probabilitiy that you find yourself at state one at time 1 trillion would be approximately 2/7.

The probability that you find yourself at state one at time 2 trillions is again approximately 2/7.

So essentially what we have here is the probability of being in that state one settles in a steady value.

That's what the steady state convergence means.

It's convergence of probabilities, not convergence of the process itself.

Again, the process will keep jumping back and forth, but the steady state probability will settle for a given value here in one, that will be 2/7, and the steady state probability in being in two will settle to 5/7.

And finally in this example, and more generally when we have a single class and no periodicity, the initial state does not matter.